

4. Find the r^{th} moment about the origin for the RV with pdf $f(x) = kx^2 e^{-x}$, $x > 0$. Hence find first moments about the origin

Solution:

To find 'k'

By the definition, we've $\int_{-\infty}^{\infty} f(x) dx = 1$.

$$\Rightarrow k \int_0^{\infty} x^2 e^{-x} dx = 1.$$

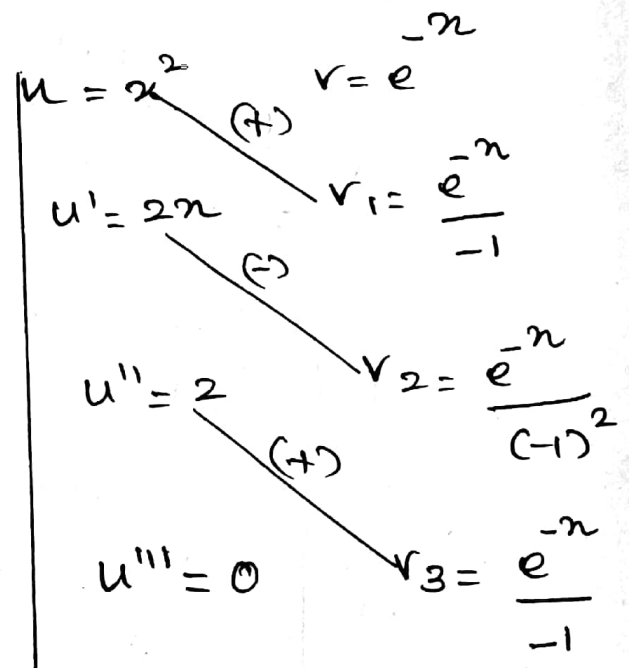
$$\Rightarrow k \left\{ -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} \right\}_0^{\infty} = 1.$$

$$\Rightarrow k \{ 0 - (-2) \} = 1.$$

$$2k = 1$$

$$k = \frac{1}{2}$$

$$\therefore f(x) = \frac{1}{2} x^2 e^{-x}, x > 0$$



r^{th} Moment about the origin

$$M_r' = \int_{-\infty}^{\infty} x^r f(x) dx = \frac{1}{2} \int_0^{\infty} x^r x^2 e^{-x} dx.$$

$$= \frac{1}{2} \int_0^{\infty} x^{r+2} e^{-x} dx.$$

$$M_r' = \frac{1}{2} \int_0^{\infty} e^{-x} x^{r+2} dx$$

$$= \frac{1}{2} \int_0^{\infty} e^{-x} x^{(r+3)-1} dx.$$

$$M_r' = \frac{1}{2} \Gamma(r+3).$$

Since by the definition of Gamma function, we've

$$\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx.$$

$$\text{also } \Gamma(n) = (n-1)!$$

$$\Gamma(n+1) = n!$$

To find the moment about the origin

$$M_r' = \frac{1}{2} \Gamma(r+3) \text{ --- (1)}$$

When $r=1$, (1) $\Rightarrow M_1' = \frac{1}{2} \Gamma(4)$

$$= \frac{1}{2} (3)! = \frac{1}{2} (6).$$

$$M_1' = 3.$$

When $r=2$, (1) $\Rightarrow M_2' = \frac{1}{2} \Gamma(5) = \frac{1}{2} (4)!$

$$= \frac{1}{2} (24).$$

$$M_2' = 12$$

When $r=3$, (1) $\Rightarrow M_3' = \frac{1}{2} \Gamma(6) = \frac{1}{2} (5!).$

$$= \frac{1}{2} (120).$$